



$$\frac{d^2x}{dt^2} + 1 \frac{dx}{dt} + 0.5x = 0.1$$



BIPOLAR ANALOG COMPUTATION

AD

NEW
CONCEPTS

BULLETIN NO.

1



Applied Dynamics, Inc.

BIPOLAR ANALOG COMPUTATION

by

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1. INTRODUCTION TO BIPOLAR AMPLIFIERS

One of the most significant developments in analog computer technology in the past several years has been the introduction of all solid-state nonlinear computing elements. These include quarter-square multipliers as well as fixed and variable diode function generators. Each of these nonlinear components requires both polarities of input voltages, so that normally many unity-gain inverting amplifiers, either terminated on the patchboard or permanently assigned to nonlinear elements, are required in solving large nonlinear problems. An important feature of the AD-256 computer system is the use of bipolar operational amplifiers, i.e., amplifiers with both output polarities terminated on the patchboard. This is accomplished by permanently connecting a precision unity-gain stabilized amplifier to the output of every integrating and summing amplifier on the patchboard. With bipolar amplifiers the bipolar inputs required for all nonlinear elements are always available and can easily be patched using two single patchcords or special twin patchcords. In addition, with bipolar amplifiers the overall programming of any problem is greatly simplified, since both polarities of every problem variable are always available.

To illustrate the programming ease of bipolar amplifiers and to introduce the recommended symbols, the bipolar circuit diagrams for several typical problems have been prepared in the following sections. In addition there is presented the basic AD concept of viewing nonlinear computing components as nonlinear input gains to amplifiers. This concept allows simple and accurate programming of nonlinear problems without permanently committing output amplifiers to each nonlinear element.

2. A SIMPLE MASS-SPRING-DAMPER PROBLEM

Consider first the mass-spring damper problem in Figure 1. Note the symbolism for the bipolar integrators. The gain of each amplifier input is indicated above the input line, and is referred to a 1 microfarad feedback capacitor for integrators (1 megohm feedback resistor for summers). The integrator gain referred to a 1 megohm input resistor is indicated in the center of the rectangle behind each integrator triangle (1 for a 1 mfd feedback capacitor, 10 for a 0.1 mfd capacitor).

Note that both output polarities are shown for integrators A1 and A2, and that the upper output lead is always the negative sign of the output variable. In the figure we have assumed that the parameters m , c , and k are all positive. However, if any of these parameters are negative, it is only necessary to reverse the input polarity into the corresponding coefficient potentiometer. Reference voltage is indicated as +1 (actually, +100 volts) in the figure.

$$m\ddot{y} = -c\dot{y} - ky + f(t)$$

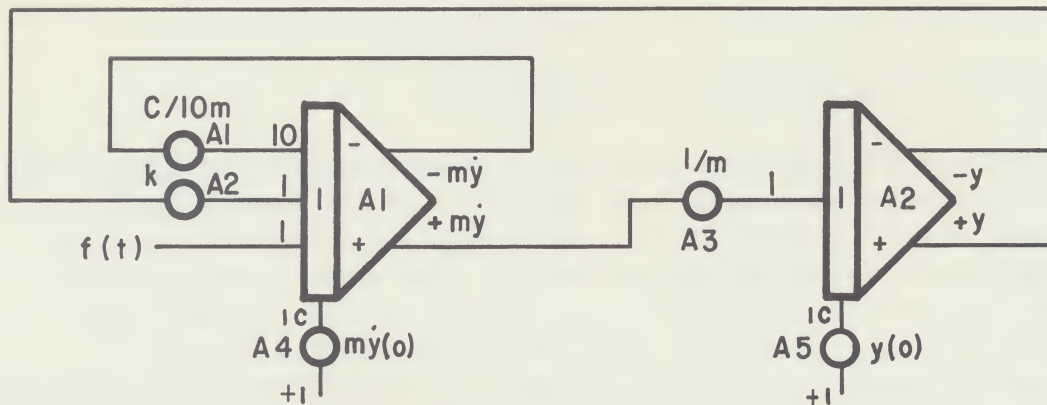


Figure 1. Bipolar circuit for mass-spring-damper problem.

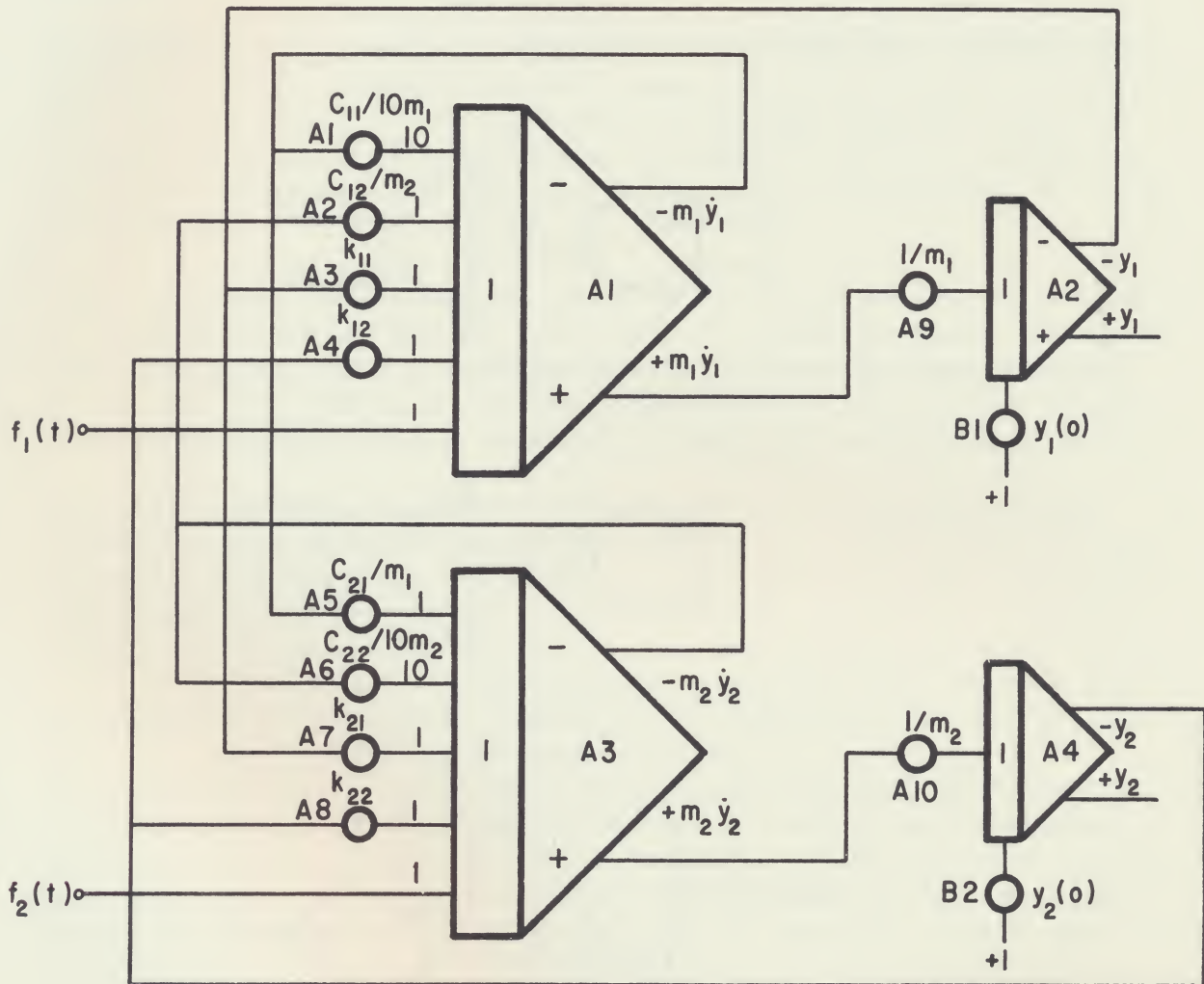
3. TWO-DEGREE-OF-FREEDOM SYSTEM

As a second example, consider the two-degree-of-freedom problem given in Figure 2. We can think of the equations as representing two mass-spring-damper systems with both spring and damper coupling between the systems. Note that only 4 bipolar integrators are needed to solve the problem, and that the polarity of any coefficient can be reversed without adding any amplifiers to the circuit. For simplicity, only the displacements y_1 and y_2 are provided with non-zero initial conditions in the figure.

4. NONLINEAR COMPUTING ELEMENTS

As stated in the introduction, bipolar amplifiers make assigned amplifiers for nonlinear elements unnecessary. Both polarities of input variables are always available and a patchboard amplifier can be used to terminate the element. Figure 3 summarizes the required patching for typical nonlinear elements. In all of these circuits full-scale computer reference voltage (normally +100 volts) is considered to be unity. Figure 3a shows the connections for a multiplier. The multiplier is terminated in the summing junction with a 0.1 megohm feedback, as

indicated with the .1 gain designation for the amplifier. The multiplier gain referred to a 1 megohm impedance is 10, as indicated inside the multiplier symbol. Thus we can think of the multiplier as an input impedance with gain $10X_1$ to which an input voltage X_2 is applied.

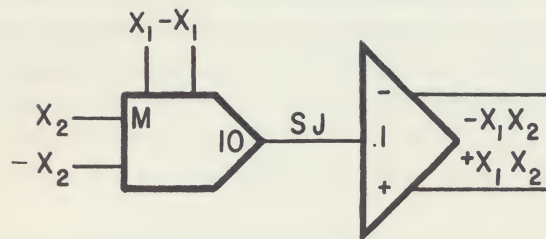


$$m_1 \ddot{y}_1 = -c_{11} \dot{y}_1 - c_{12} \dot{y}_2 - k_{11} y_1 - k_{12} y_2 + f_1(t)$$

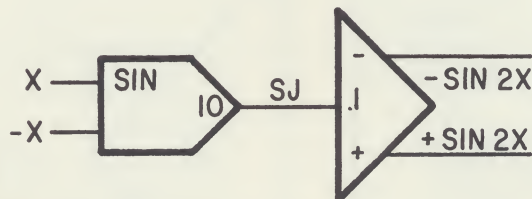
$$m_2 \ddot{y}_2 = -c_{21} \dot{y}_1 - c_{22} \dot{y}_2 - k_{21} y_1 - k_{22} y_2 + f_2(t)$$

Figure 2. Bipolar circuit for two-degree-of freedom system.

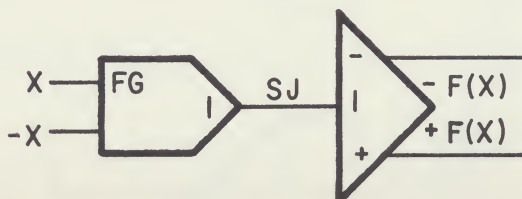
Similarly, Figure 3b shows the connections for a fixed sine DFG. As in the case of the multiplier, the fixed DFG is terminated in the summing junction of an amplifier with a 0.1 megohm feedback resistor. On the other hand, the variable



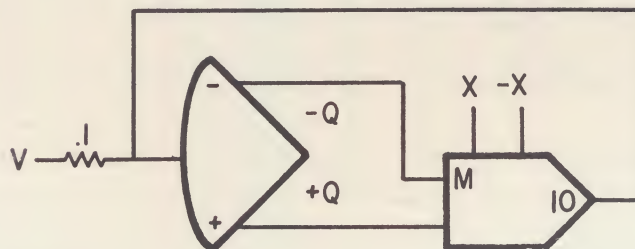
(a) Multiplier with bipolar output amplifier



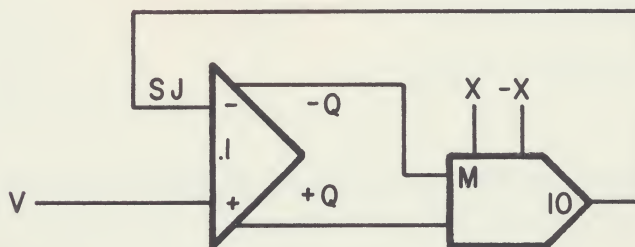
(b) Fixed Sine DFG with bipolar output amplifier



(c) Variable DFG with bipolar output amplifier



(d) Divider circuit for $Z = V/X$, $X > 0$



(e) Divider circuit for $Q = V/(1 + X)$, $-1 < X < 1$

Figure 3. Nonlinear Circuits with Bipolar Output Amplifiers.

DFG of Figure 3 terminates in the summing junction of an amplifier with 1 megohm feedback. Both the amplifier and DFG gains are therefore designated as 1.

It should be noted that the outputs of any number of nonlinear or linear elements can be summed in a single output amplifier. This will be illustrated in the next section.

The divider circuit of Figure 3d is best understood by again considering the multiplier as a variable gain at the amplifier input, where here the gain is given by $10X$. With the multiplier input connected to the high-gain amplifier output, the amplifier has a gain of $1/10X$. Therefore the voltage V with the 0.1 megohm input resistor experiences a net gain of 10 ($1/10X$) = $1/X$. Thus the output $Q = V/X$ as required.

Figure 3e shows a variation of this circuit for $Q = V/(1+X)$ where $-1 < X < 1$. Here the amplifier is not high gain but is left with a 0.1 megohm feedback. The multiplier output is fed back to the summing junction. Clearly the amplifier output $Q = V - QX$, or $Q = V/(1+X)$ as required. This circuit is more accurate than the one of Figure 3d since all four multiplier quadrants are used instead of just two.

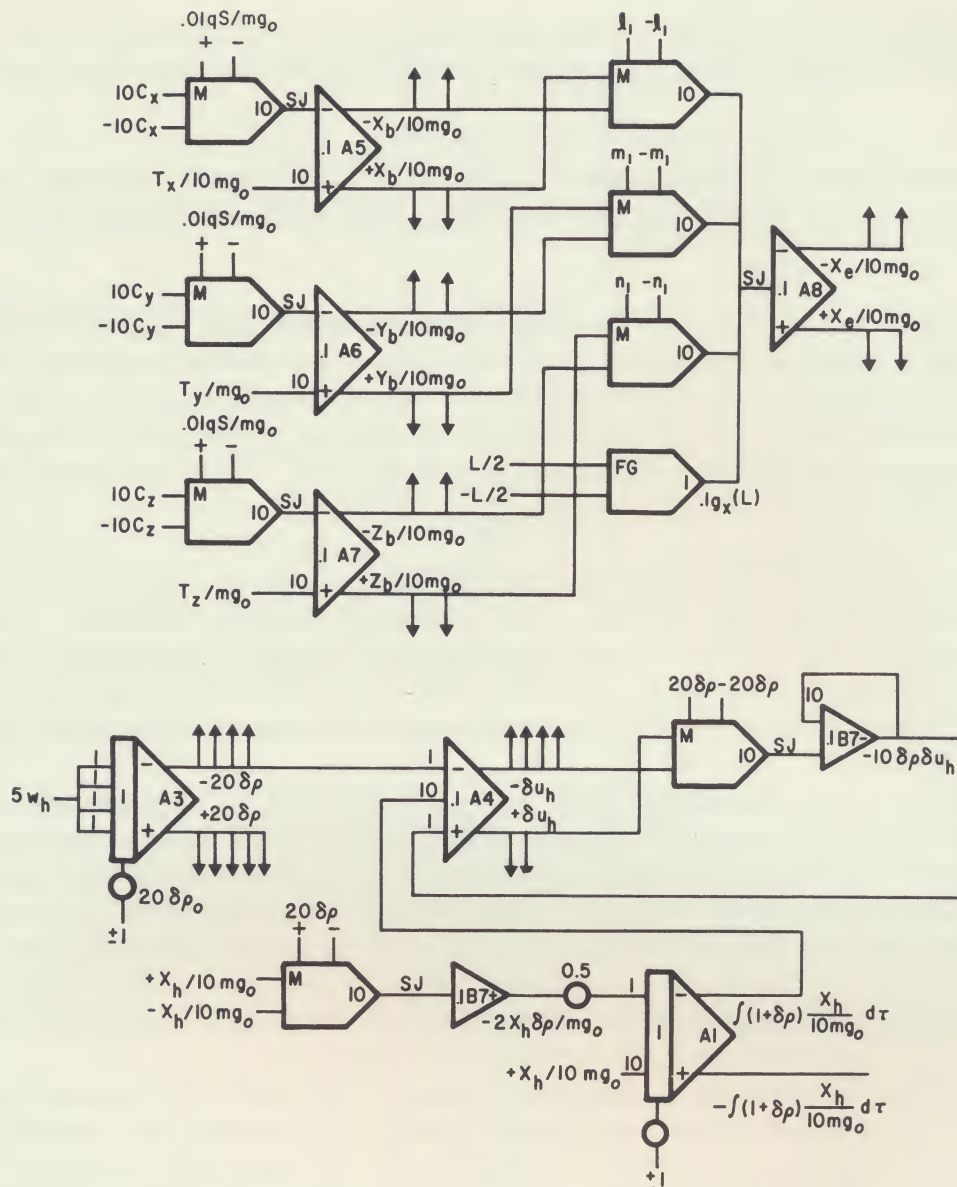
It should be noted that all summers in the AD-256 have 0.1 megohm feedback resistors in their normal patchboard configuration. This allows convenient termination of multipliers and fixed DFG's and also improves amplifier bandwidth. These amplifiers can be converted either to high-gain amplifiers or 1 megohm feedback amplifiers by inserting a shunt plug in the appropriate patchboard holes.

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5. EXAMPLE OF NONLINEAR COMPUTATION WITH BIPOLAR AMPLIFIERS

As an example of the use of nonlinear computing elements with bipolar amplifiers, consider the circuit shown in Figure 4. Here we are solving the translational equations of motion of an orbital vehicle of mass m which is subjected to central-force field gravity forces, oblate-earth gravity-correction forces, aerodynamic forces, and rocket-thrust forces T_x , T_y , and T_z^* . Additional circuit inputs are dynamic-pressure acceleration $\pm qS/mg_0$, aerodynamic force coefficients $\pm C_x$, $\pm C_y$, and $\pm C_z$, 9 bipolar direction cosines, and latitude in radians, $\pm L/2$. Bipolar outputs from the circuit include dimensionless horizontal velocity $\pm \delta u_h$, vertical velocity $\pm 5 w_h$, radial variation $\pm 20 \delta \rho$; and $\pm \sin \psi_h$ and $\pm \cos \psi_h$, where ψ_h is the flight-path heading angle. The various equations are given in the figure. As in the previous section, scaling is such that computer reference voltage is considered to be unity. Component requirements include the following:

*For a detailed discussion of these equations, see L. E. Fogarty and R. M. Howe, "Flight Simulation of Orbital and Re-entry Vehicles" - IRE Transactions on Electronic Computer Vol. EC-11 Aug. 1962.



$$\begin{aligned}
 X_b &= qSC_x + T_x & X_e &= X_b \ell_1 + Y_b m_1 + Z_b n_1 + mg_x(L) & X_h &= X_e \cos \psi_h + Y_e \sin \psi_h \\
 Y_b &= qSC_y + T_y & Y_e &= X_b \ell_2 + Y_b m_2 + Z_b n_2 & Y_h &= -X_e \sin \psi_h + Y_e \cos \psi_h \\
 Z_b &= qSC_z + T_z & Z_e &= X_b \ell_3 + Y_b m_3 + Z_b n_3 + mg_z(L) & Z_h &= Z_e
 \end{aligned}$$

$$\begin{aligned}
 \delta u_h &= \frac{1}{1 + \delta \rho} \left[\int (1 + \delta \rho) \frac{X_h}{mg_o} d\tau - \delta \rho \right], & \frac{dw_h}{d\tau} &= -\frac{\frac{\delta \rho}{1 + \delta \rho} + 2\delta u_h + (\delta u_h)^2}{1 + \delta \rho} + \frac{Z_h}{mg_o}, \\
 \frac{d\delta \rho}{d\tau} &= -w_h, & \frac{d\psi_h}{d\tau} &= \frac{Y_h}{mg_o(1 + \delta u_h)} + (\dot{\lambda} + r_n) \sin L
 \end{aligned}$$

Figure 4. Circuit for orbital equations of motion using Bipolar amplifiers.

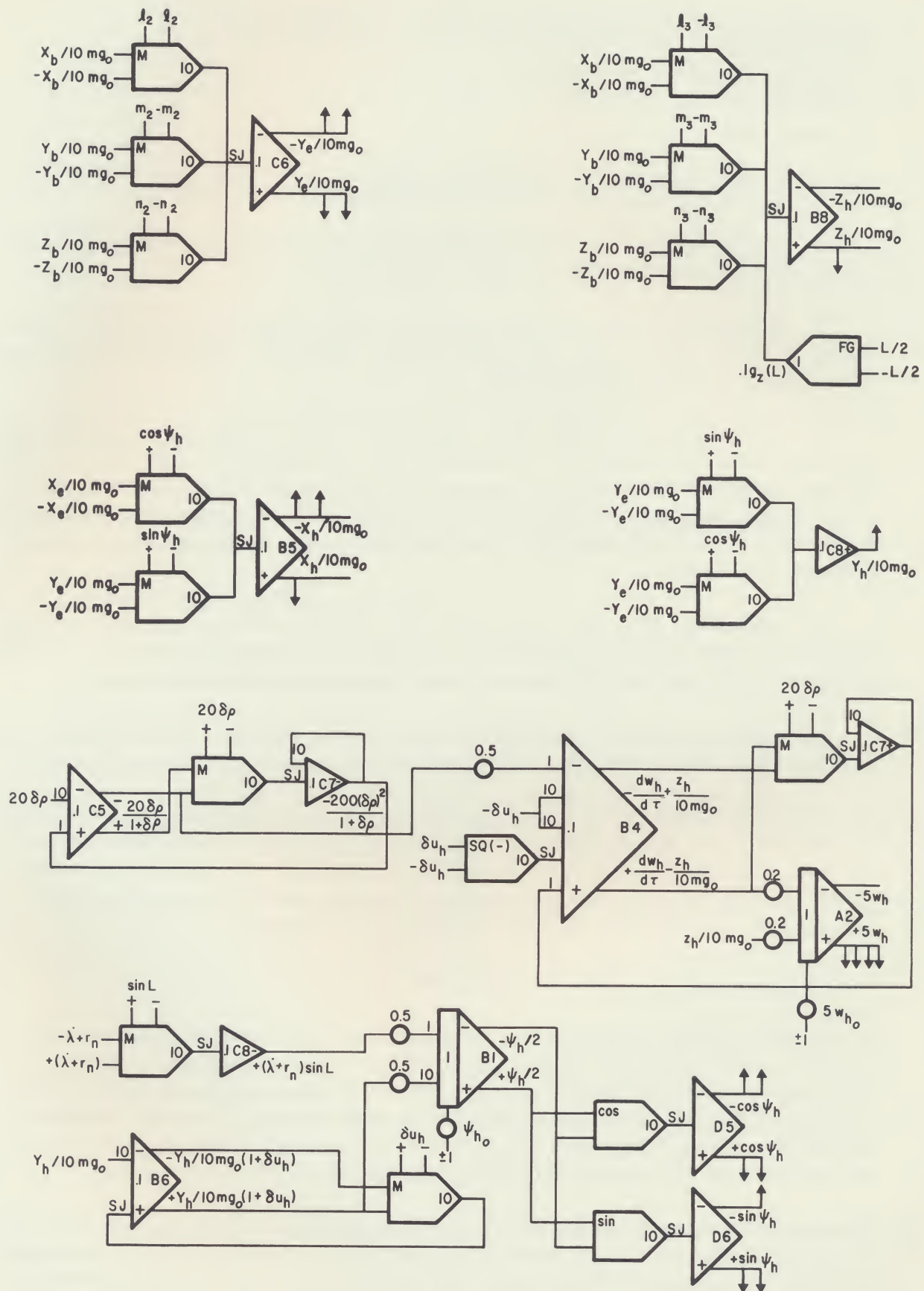


Figure 4. (Continued)

20 bipolar amplifiers (3 bipolars converted to 6 conventional)
22 multipliers
2 variable DFG's
1 sine DFG
1 cosine DFG

Let us list some of the high-points of the circuit diagram of Figure 4.

1. Amplifiers A5, A6, and A7 at the upper left of the figure terminate multipliers and also sum an additional input, respectively.

2. Amplifiers A8 and B8 each terminate three multipliers and a DFG, producing an output equal to the sum of the inputs. Similarly, amplifiers C6, B5 and B6 terminate several products each.

3. In the lower left of the figure amplifier A4 implements the divider circuit described earlier in Figure 3e, except that here the multiplier output is fed back through amplifier B7-in order to attenuate the product by a factor of 20 (factor of 2 gain reduction in B7- because of the two paralleled 0.1 megohm feedback resistors, factor of 10 reduction through the 1 megohm input to A4, which has an 0.1 megohm feedback resistor).

4. Amplifiers B7-, B7+, C7-, C7+, C8-, and C8+ are conventional summing amplifiers which terminate multipliers. In the AD-256 these amplifiers, normally bipolar, can be separated in pairs as shown using shunt-activated relays.

It is suggested that the reader may wish to trace through several of the circuits of Figure 4 in some detail in order to confirm the method of nonlinear elements.

Note that all bipolar amplifiers in Figure 4 have in general been programmed so that the minus (upper) output terminal of the amplifier is the negative sign of the output variable, and the positive (lower) terminal is the positive sign of the output variable. This is always possible to achieve since both input polarities are generally available to any bipolar summer or integrator.

6. THE COST OF BIPOLAR AMPLIFIERS

Granted an obvious improvement in programming ease, one invariably asks the question, "How much extra does it cost to have bipolar amplifiers?" For a fully-expanded AD-256 the 128 summing and integrating amplifiers require 128 precision inverting amplifiers to make all outputs bipolar. These 128 additional amplifiers cost approximately \$20,000, which represents less than 10 percent of the overall cost of a typical fully-expanded AD-256.

But this is far from the whole story. The availability of both output polarities for every problem variable eliminates completely the need for permanently-assigned amplifiers for nonlinear components such as multipliers and function generators. When one considers that over 160 such nonlinear elements can be terminated on the AD-256 problem board, the extent of this saving becomes evident. Many

current manufacturers of analog equipment commit at least one inverting amplifier to each multiplier or DFG, which means that over 160 such committed amplifiers could be required in the AD-256 if such a philosophy were followed. These would actually cost more than the 128 permanent inverters used for bipolar operation in the AD-256.

Furthermore, there are invariably a large number of inverting amplifiers required in almost any analog program just for the purpose of obtaining the correct polarity for input variables to summers. It should also be pointed out that up to 32 of the AD-256 inverters can be converted to conventional summers with patchboard-controlled relays. This makes a total of 160 general purpose summing and integrating amplifiers in a fully expanded AD-256, with 96 amplifiers permanently connected as inverters. Thus at the programmer's option he can have up to 62.5% of his 256 amplifiers used for summing and integrating, with 37.5% used as permanent inverters. Even with computers having committed amplifiers with nonlinear components, this summer and integrator-to-inverter ratio is typical of large problems.

In the light of these facts it seems clear that bipolar amplifiers actually save in overall computer cost.

7. ACCURACY CONSIDERATIONS

The inverting amplifiers used for the AD-256 bipolar amplifiers are stabilized amplifiers identical in every way with the summing and integrating amplifiers. The feedback and input resistors are matched to better than 0.002% and are located directly behind the patchboard along with all other computing resistors and capacitors. This prevents any significant errors due to voltage drop in the inverter output leads. As in the case of all computing resistors in the AD-256, the inverting-amplifier resistors are adjustable with trimmers so that any long-term wire-wound resistor drift can easily be compensated.

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The inverting amplifiers use 100K input and feedback resistors in order to provide better dynamic accuracy. Typical inverter bandwidth is well over 100 kc, and typical phase shift at 100 cps is less than 0.02 degrees. The superior static and dynamic performance of the bipolar inverting amplifiers insures negligible accuracy loss in using any inverted voltage signal. This is particularly significant when compared with conventional computers having assigned inverters to nonlinear components, where often a voltage signal is inverted twice before it is applied to the actual nonlinear component (once in an inverter on the problem board, a second time in the inverter committed to the nonlinear component).

8. PROGRAMMING CONVENIENCE

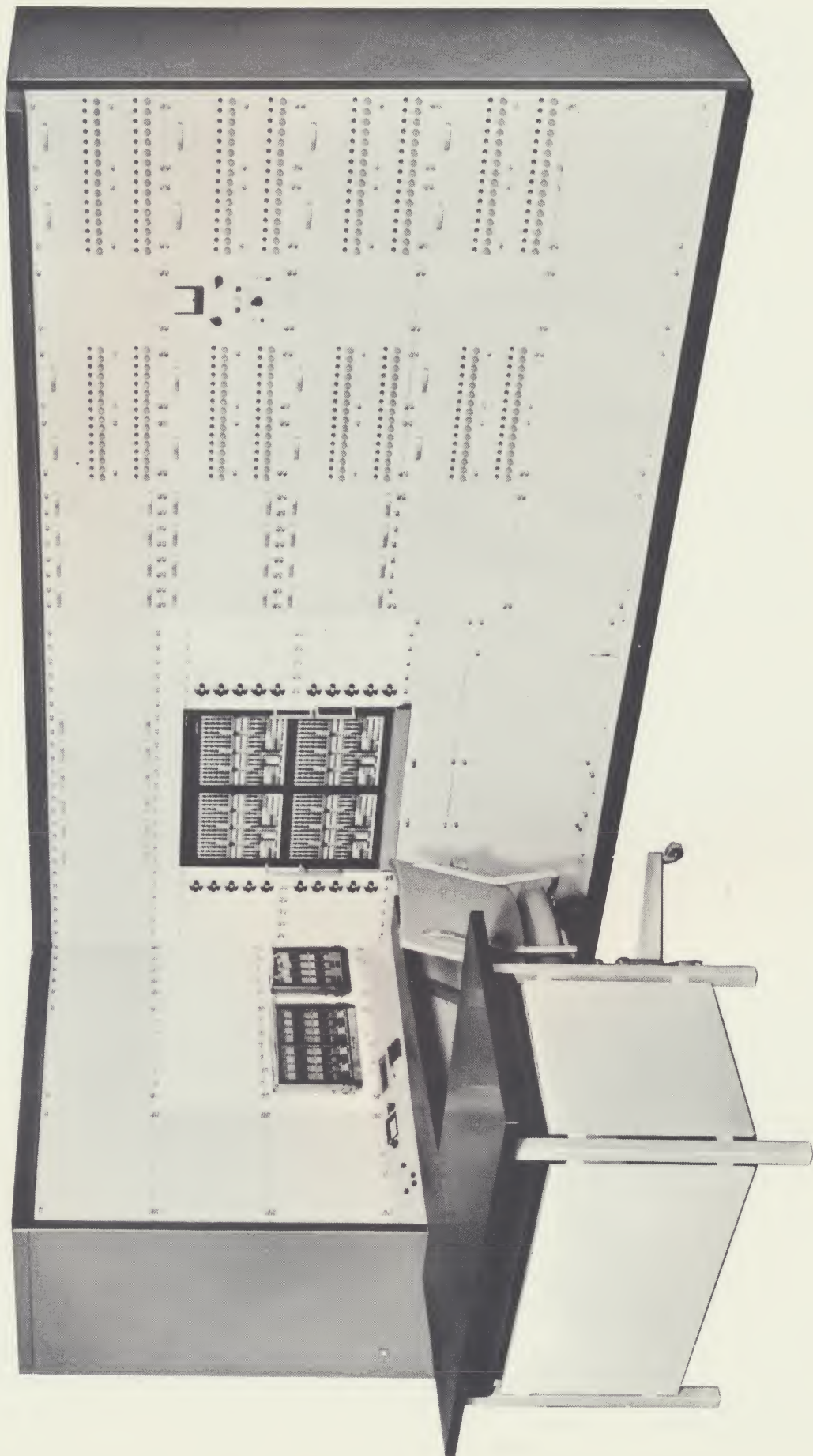
The programming convenience of bipolar amplifiers has already been emphasized. Every bipolar amplifier is terminated on the patchboard with alternate black and red holes in a vertical row representing, respectively, negative and positive output polarities. Not only is either polarity always available for every problem variable, but to change polarity, as in the case of a coefficient set on a poten-

tiometer, it is only necessary to move the patchcord from one output hole to the adjacent hole representing opposite output polarity. Furthermore, special patchcords with a twin plug on each end allow convenient connection of bipolar outputs to nonlinear elements using a single patching step.

Another significant improvement in programming convenience using bipolar amplifiers is the saving in patching connections, since it is no longer necessary to connect an amplifier output to the input of an inverter in order to reverse its sign. Incidentally, this also saves two required patchboard holes per inverter (one output hole for the conventional amplifier, one input hole for the inverter).

As pointed out earlier, 32 of the bipolar inverters in the AD-256 can be converted in pairs to general-purpose summers using bottle plugs. Normally this feature would not be used by the programmer in order that he maintain the extreme convenience of bipolar outputs. It would only be used when the programmer is pushing the computing capability of the AD-256 to its ultimate.

It is felt that the introduction of bipolar amplifiers represents a significant improvement in analog-computer programming.



NEW AD-256 Bipolar Analog Computer



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